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ONE-IMPULSE MINIMUM-FUEL NONCOPLANAR INTERORBITAL FLIGHT

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[Article by S. N. Kirpichnikov]

[Text] The problem of finding one-pulse minimum-fuel flights between given noncoplanar boundary orbits both with free time and with regard to the time of motion along the orbits is studied. Motion occurs in a central Newtonian gravity field and all the orbits are Keplerian. The characteristic velocity of the initial pulse is minimized. Unlike the problem of interorbital crossing (see survey [1]), a trajectory that only intersects the final orbit, beginning at the initial orbit, must be constructed in the problem of interorbital pulse flight, since the velocity-equalizing pulse is not reached at the finish point. We emphasize that, unlike a number of papers ([2-5] and so on), neither the take-off or finishing point in boundary orbits is fixed and neither is subject to determination together with the intermediate orbit of the flight.

The conditions of flight optimality in the planes of boundary orbits are found below. Approximate optimal solutions in the practically important case of slightly elliptical boundary orbits, the angle between the planes of which is small, are found. The proposed method permitted very simple determination of the branch of steady-state solutions and study of their qualitative structure. These solutions can be used very effectively as initial approximations when finding the orbits of real interplanetary flights numerically.

1. Mathematical postulation of problem. General conclusions. Let us determine Keplerian orbits by the following elements:

$$p=[a(1-e^2)]^{-1/2},\ q=e[a(1-e^2)]^{-1/2},\ w,\ i,\ \Omega,\ T, \qquad (1)$$

where a, e, w, i, Ω, and T are the major semiaxis, eccentricity, longitude of the pericenter, inclination, longitude of some previously

fixed (ascending or descending) orbital node, and the moment of passage through the epicenter, respectively. If the ascending node is fixed, then $i \in [0, \pi]$, and if it is descending, then $i \in [-\pi, 0]$. Motion along the orbit is direct at $|i| \leq \pi/2$ and reciprocal at $|i| > \pi/2$. If the given orbit is circular, then T is the moment of passage through a point with fixed value w of longitude in the orbit.

Let us introduce the following notations: p_1, q_1, w_1, i_1, Ω_1, and T_1 and p_2, q_2, w_2, i_2, Ω_2, and T_2 are the given elements of the initial (subscript 1) and final (subscript 2) orbits, p, q, w, i, Ω, and T are elements of the intermediate orbit of the flight, u_1 and $\tilde{u}_1$ are the longitudes of the take-off point in initial and intermediate orbits, $\tilde{u}_2$ and u_2 are the longitudes of the finish point in intermediate and final orbits, and t_1 and t_2 and r_1 and r_2 are moments of time and polar radii of the take-off and finish points, respectively. Let us take the plane of the initial orbit as the plane of reference, and let us take the direction of motion in it as the direction of positive reading of longitudes. Let us select as fixed points: an ascending node for the final orbit and the node at the take-off point for the intermediate orbit; then always

$$i_1=0,\ i_2\geqslant 0,\ \Omega=\tilde{u}_1=u_1. \tag{2}$$

The conditions of continuity of radii r_1 and r_2 at the take-off and finish points yield

$$\varphi_1=p^2+pq\cos(u_1-w)-p_1^2-p_1q_1\cos(u_1-w_1)=0, \tag{3}$$

$$\varphi_2=p^2+pq\cos(\tilde{u}_2-w)-p_2^2-p_2q_2\cos(u_2-w_2)=0. \tag{4}$$

Using formulas of spherical trigonometry, we easily find

$$\varphi_3=\cos(\tilde{u}_2-u_1)-\cos(u_2-\Omega_2)\cos(\Omega_2-u_1)+\sin(u_2-\Omega_2)\sin(\Omega_2-u_1)\cos i_2=0, \tag{5}$$

$$\varphi_4=\sin(\tilde{u}_2-u_1)\sin i-\sin(u_2-\Omega_2)\sin i_2=0. \tag{6}$$

For the characteristic velocity of the initial pulse, we have

$$\Delta U = K\Delta V,\ \Delta V = \{q_1^2 + 3p_1^2 + q^2 - p^2 - 2p_1^3/p - 2qq_1 \cos(w_1 - w) - 2(p_1 - p)^2 p^{-1} q_1 \cos(u_1 - w_1) + 4p_1 p^{-1}[p_1 + q_1 \cos(u_1 - w_1)]^2 \times \sin^2(i/2)\}^{1/2}, \tag{7}$$

where K is the gravitational parameter of the central body.

Functions $g = (\Delta V)^2/2$ were minimized on the set of variables

$$p,\ q,\ w,\ u_1,\ \tilde{u}_2,\ u_2,\ i, \tag{8}$$

which are dependent and which satisfy connections (3)-(6) in problems with free time. The condition of coincidence of the time of motion to the finish point is added to these connections in the problems with regard to the time of motion along the orbits:

$$\varphi_5 = \psi_1(u_1,\ p_1,\ q_1,\ w_1) + \psi(u_1,\ \tilde{u}_2,\ p,\ q,\ w) - \psi_2(u_2,\ p_2,\ q_2,\ w_2) - K(T_2 - T_1) = 0, \tag{9}$$

where functions ψ, ψ_1, and ψ_2 are determined by formulas (3) and (4) of [6]. Let us use the Lagrange method when compiling systems of the necessary conditions of the extreme value. The Lagrange multipliers, corresponding to connections (3)-(6) and (9), will be denoted by λ_1, ..., λ_5, respectively.

Theorem 1. One-impulse minimum-fuel flight between coplanar boundary orbits occurs in the plane of these orbits.

The following lemma is valid for noncoplanar boundary orbits ($\sin i_2 \neq 0$).

Lemma. 1. Fixed flights in the plane of the initial orbit and only they are characterized by equality $\sin(u_2 - \Omega_2) = 0$. 2. Fixed flights in the plane of the final orbit and only they are characterized by conditions: $\sin(u_1 - \Omega_2) = 0$, $\sin(u_2 - \Omega_2) \neq 0$. 3. The orbit of a one-impulse flight whose plane does not coincide with the plane of either the initial or final orbit may not touch either the initial or final orbits.

2. One-impulse minimum-fuel flight with free time between noncoplanar nonintersecting elliptical orbits. The direction of motion in the final orbit plays no role; therefore, let us conditionally assume that it is direct. The problem under consideration in coplanar postulation has been studied in [6-9]; therefore, we assume that $0 < i_2 \leq \pi/2$.

The problem of the conditions of optimality of flights in the planes of the initial and final orbits is of important practical significance. We find the following three theorems from analysis of a system of necessary conditions.

Theorem 2. A one-impulse flight in the plane of the initial orbit, which ends at one of the nodal points of the final orbit, is globally optimal with respect to fuel in the class of limited one-impulse flights between noncoplanar circular orbits.

Theorem 3. Only the two following types of fixed orbits of one-impulse flights in the plane of the initial orbit ($i = 0$, $\tilde{u}_2 = u_2$, $\sin(u_2 - \Omega_2) = 0$) are possible. Fixed orbits in the coplanar problem of one-impulse minimum-fuel flight from the initial orbit to a fixed point (one of the nodal points of the final orbit), which satisfy the additional condition

$$p_2 q_2 \sin(\Omega_2 - w_2) - pq \sin(\Omega_2 - w) \cos i_2 = 0. \quad (10)$$

2. When the nodal line of the final orbit coincides with the apsidal line of the initial orbit

$$q_1 \sin(\Omega_2 - w_1) = 0 \quad (11)$$

there are two Hohmann orbits with apsidal tangential pulses. The take-off and finish points for each of them are the apses of the flight orbit and they lie on the nodal line of the final orbit; moreover, the take-off point is one of the apses of the initial orbit, while the finish point is the nodal point of the final orbit.

Theorem 4. Only the two following types of fixed orbits of one-impulse flights in the plane of the final orbit are possible ($i = i_2$, $\sin(u_1 - \Omega_2) = 0$, $\tilde{u}_2 = 2$). 1. Fixed orbits in the coplanar problem of one-impulse flight from a fixed point (one of the points of the intersection of the initial orbit with the plane of the final orbit) to the final orbit upon minimization of function (7) with $i = i_2 = \text{const}$, which satisfy the additional condition of contact of the flight orbit with the final orbit at the finish point and the condition $\sin(u_2 - u_1) \neq 0$. 2. Orbits determined by relations (3) and (4) and by the equalities

$$q \sin(u_1 - w) = q_1 \sin(u_1 - w_1), \quad p \cos i_2 = p_1, \quad p \cos(u_2 - u_1) + q \cos(u_2 - w_1) = 0. \quad (12)$$

Here the initial pulse is orthogonal to the orbital plane of the flight.

The plane of a one-impulse minimum-fuel flight generally does not coincide with the plane of either the initial or final orbits. Indeed, according to theorems 3 and 4, fixed flights in the planes of boundary orbits coincide to some hypersurfaces in the space of elements of the boundary orbits. Let us also note the following general result.

Theorem 5. A tangential initial impulse may be optimal only if condition (11) is fulfilled for fixed orbits of second type, determined by theorem 2. These orbits lie in the plane of the initial orbit and, unlike other fixed orbits, are independent of inclination i_2.

Using the method of [10], it is easy to construct the approximate fixed and optimal solutions in cases when $q_1 \sin(\Omega_2 - w_1)$ is small, and also when $q_1 q_2 \sin(w_2 - w_1)$ and i_2 are small. Zero approximation fixed orbits were found above (i_2 is finite) and in [7, 8] (i_2 is small).

Let us select as an example the case of slightly elliptical boundary orbits of small mutual inclination i_2. (A similar problem for a two-impulse flight is solved in [11]). It turned out when studying the system of required conditions that the second-order branching point corresponds to the zero approximation optimal orbit, and the resulting series is arranged by powers of $\sqrt{i_2}$, q_1, and q_2. Let us introduce in this regard the small parameter ϵ as follows:

$$i_2 = i_2''\varepsilon^2,\ i_2'' \neq 0,\ q_j = q_j'\varepsilon + q_j''\varepsilon^2 + \dots,\ j = 1,\ 2. \tag{13}$$

Let us find the solution of the system of necessary conditions in the form of a series by powers of ϵ. Let us retain the ordinary notations for all zero approximation parameters. Let us denote by primes the coefficients of a desired series at ϵ and let us denote by double primes the coefficients at ϵ^2. A Hohmann ellipse, determined by the following relations, is minimum-fuel optimal in zero approximation ($\epsilon = 0$) when flying between coplanar circular orbits of radii $r_1 = p^{-2}{}_1$ to $r_2 = p^{-2}{}_2$

$$p = \sqrt{(p_1^2 + p_2^2)/2},\ q = |p_1^2 - p_2^2| / \sqrt{2(p_1^2 + p_2^2)},\ \Delta V = p_1 |p_1 - p| / p, \tag{14}$$

$$i = 0,\ \tilde{u}_2 = u_2 = u_1 + \pi,\ w = u_1 + \pi(1 - \delta)/2,\ \delta = \operatorname{sign}(r_2 - r_1), \tag{15}$$

$$\lambda_1 = (p_2^4 - p_1^3 p) / (4p^4),\ \lambda_2 = p_1^3 (p_1 - p)/(4p^4), \tag{16}$$

and the value of u_1 and multipliers λ_3 and λ_4 are arbitrary.

Two types of steady-state solutions are possible at sufficiently small $\epsilon > 0$.

Solutions nontrivial in first approximation. First let $(q_1')^2+(q_2')^2>0$. For the desired solutions that determine the function g to be minimized with accuracy up to order ϵ^2 inclusively, we have

$$\operatorname{tg} u_1 = \frac{p_1[2(p_1-p)+\lambda_1 p]\, q_1' \sin w_1 - p_2 p \lambda_2 \sin w_2}{p_1[2(p_1-p)+\lambda_1 p]\, q_1' \cos w_1 - p_2 p \lambda_2 \cos w_2}, \tag{17}$$

$$(i')^4 p_1^3 (\lambda_1\lambda_2 p^3)^{-1} + i'(A_2 - A_1)\, i_2'' \sin(u_2-\Omega_2) + (i_2'')^2 \sin^2(u_2-\Omega_2) = 0, \tag{18}$$

$$u_1' = w' + B_1,\ \ddot{u}_2' = u_2' = w' + B_2,\quad B_j = A_j - \frac{\delta p_1^3 (i')^3}{\lambda_j p^2 q i_2'' \sin(u_2-\Omega_2)},\ j=1,2, \tag{19}$$

$$A_1 = \frac{\delta[(p-p_1)^2 + \lambda_1 p p_1]\, q_1' \sin(u_1-w_1)}{\lambda_1 p^2 q},\quad A_2 = -\frac{\delta p_2 q_2' \sin(u_2-w_2)}{pq}. \tag{20}$$

Coefficient w' is arbitrary within the accepted accuracy and coefficients p', p'', q', q'', $\Delta V'$, and $\Delta V''$ are not presented here and below, since accurate solutions of the corresponding variables can easily be found by formulas (3), (4), and (7). Equation (17) determines the two values of angle u_1, differing by π, and the characteristic velocity assumes the smaller value if the signs of the numerator and denominator of the right side of (17) coincide with those of $\sin u_1$ and $\cos u_1$, respectively. Equation (18) has two real roots: the take-off occurs for one root ($i' > 0$) in the ascending node of the flight orbit, and that for the other ($i' < 0$) occurs in the descending node. If it turned out upon solution of equation (17) that $\sin(u_2 - \Omega_2) = 0$, one should turn to the limit at $\sin(u_2 - \Omega_2) \to 0$ to find the corresponding solution in formulas (18)-(20).

Now let $q'_1 = 0$ and $q'_2 = 0$, i.e., the mutual inclination of boundary orbits and their eccentricities are values of the same order of smallness. This case is important in study of interplanetary flights. We have for angle u_1

$$\begin{aligned}&[2(p_1-p)+\lambda_1 p]\, p_1 p^{-1} q_1'' \sin(u_1-w_1) - \lambda_2 p_2 q_2'' \sin(u_1-w_2) + \\ &+ \operatorname{sign}[\sin(u_1-\Omega_2)] \sqrt{|\lambda_1\lambda_2|\, p p_1^3}\; i_2'' \cos(u_1-\Omega_2) = 0,\quad \sin(u_1-\Omega_2) \neq 0.\end{aligned} \tag{21}$$

The remaining coefficients are found, as above, by formulas (18)-(20).

Solutions trivial in first approximation. They exist if

$$q_1' \sin(\Omega_2 - w_1) = 0,\ q_2' \sin(\Omega_2 - w_2) = 0 \tag{22}$$

(specifically, at $q'_1 = q'_2 = 0$), and they are determined by the relations

$$\sin(u_2 - \Omega_2) = 0,\ i' = u_1' = \tilde{u}_2' = u_2' = w' = p' = q' = \Delta V' = 0. \tag{23}$$

To characterize this branch more accurately, let us present the expressions of the coefficients at ϵ^2 for all angular variables:

$$i'' = \frac{(C_2 - C_1) p^2 \lambda_4 \lambda_1 \lambda_2}{\lambda_4^2 + p p_1^3 \lambda_1 \lambda_2},\ \tilde{u}_2^{\cdot} = u_2^{\cdot} = \frac{(i'')^2 p_1^3}{p \lambda_4 i_2^{\cdot} \cos(u_1 - \Omega_2)}, \tag{24}$$

$$u_j^{\cdot} = w'' + C_j + \delta \lambda_4 i''/(\lambda_j p q),\ j = 1, 2, \tag{25}$$

$$C_1 = \delta\,[(p_1 - p)^2 + \lambda_1 p p_1]\, q_1^{\cdot} \sin(u_1 - w_1) / (\lambda_1 p^2 q), \tag{26}$$

$$C_2 = \delta\,[2(p_1 - p) + \lambda_1 p]\, p_1 q_1^{\cdot} \sin(u_1 - w_1)/(\lambda_2 p^2 q), \tag{27}$$

$$\lambda_4 = \{\lambda_2 p_2 q_2^{\cdot} \sin(\Omega_2 - w_2) - [2(p_1 p^{-1} - 1) + \lambda_1]\, p_1 q_1^{\cdot} \sin(\Omega_2 - w_1)\} / i_2^{\cdot}. \tag{28}$$

If $\lambda_4^2 + p p_1^3 \lambda_1 \lambda_2 = 0$, the considered solution exists only at $C_1 = C_2$, and higher-order approximations should be used to determine coefficient i''.

Let us follow the variation of longitude u_1 of the take-off point with a decrease of parameters q_1 and q_2. If $(q'_1)^2 + (q'_2)^2 > 0$, the longitude u_1 coincides with that of the take-off point of the corresponding coplanar problem ($i_2 = 0$). If condition (22) is fulfilled, a new solution, inherent only to the noncoplanar problem, also appears simultaneously with the take-off point, located on the nodal line of the final orbit. There are generally steady-state solutions of two types at $q'_1 = q'_2 = 0$: in the first solution, longitude u_1 is determined from

equation (21) and in the second solution, it is determined from the condition $\sin(u_1 = \Omega_2) = 0$. It is easy to select the optimal solution from the derived steady-state solutions in each specific case by comparing the characteristic velocities corresponding to them.

3. One-impulse minimum-fuel flight between nocoplanar orbits with regard to time of motion. Here the inclination of the final orbit is $i_2 \in [0, \pi]$. Specifically, at $i_2 = \pi$, we have the case of coplanar boundary orbits with opposite direction of motion in them (this case was studied in [12] for two-impulse flights with free time).

Let us construct the approximate solution of the postulated extreme problem for slightly elliptical boundary orbits, the angle between planes of which is small. Let us eliminate multiloop flight orbits, assuming that $0 \leqslant \tilde{u}_2 - u_1 \leqslant 2\pi$. Let us introduce the small parameter ϵ by formulas

$$i_2=(1-\gamma)\pi/2+\gamma \dot{i}_2 \varepsilon^2,\ \gamma=\mathrm{sign}(\cos i_2),\ \ q_j=q_j' \varepsilon+\dot{q}_j \varepsilon^2+\ldots,\ j=1,\ 2. \tag{29}$$

A minimum-fuel orbit is a Hohmann ellipse, determined by relations (14)-(16), and $\lambda_5 = 0$ in zero approximation ($\epsilon = 0$) during flight between circular coplanar boundary orbits. Let w_s be the longitude of the material point in the initial orbit at the moment of joining of the material points [13], moving along boundary orbits. We have for angular variables

$$i=0,\ \tilde{u}_2=\gamma u_2+(1-\gamma)\,\Omega_2=u_1+\pi,\ w=u_1+\pi(1-\delta)/2,\ \delta=\mathrm{sign}(r_2-r_1), \tag{30}$$

$$\begin{gathered} u_1-w_s=\pi\left[(1+2n)\,\gamma r_2\sqrt{r_2}-(r_1+r_2)^{3/2}/\sqrt{8}\right]/(r_1\sqrt{r_1}- \\ -\gamma r_2\sqrt{r_2}),\ n\in\mathbf{Z}. \end{gathered} \tag{31}$$

The following theorem generalizes the corresponding confirmation of [7].

Theorem 6. The configuration, optimal for the beginning of the considered maneuver of a one-impulse flight, is unique during each period $\Pi_s=\Pi_1\Pi_2/|\Pi_1-\gamma\Pi_2|$. Angle $u_1 - w_s$ of the take-off moment, closest to joining of the points, corresponds to the value $n \in Z$, which minimizes the modulus of the right side of (31).

Let us find the steady-state solutions at $\epsilon \neq 0$ in the form of a power series ϵ, retaining the adopted notations for the coefficients of this series. Let us present solutions that determine function g with

accuracy up to order ϵ_2, inclusively. We have relations (18) and (19) for the values of $i', u_1' - w', \tilde{u}_2' - w', u_2' - w'$, in which expressions A_1 and A_2 should be replaced by $\tilde{A}_1$ and $\tilde{A}_2$, determined as follows:

$$\tilde{A}_1 = A_1 - \delta\lambda_5' (p - p_1) \lambda_1^{-1} p^{-1} q^{-1} p_1^{-4}, \ \tilde{A}_2 = \gamma A_2 + \delta\lambda_5' (p_2\gamma - p) \lambda_2^{-1} \times \times p^{-1} q^{-1} p_2^{-4}, \tag{32}$$

$$\lambda_5' = \{[2 (p_1 p^{-1} - 1) + \lambda_1] p_1 q_1' \sin (u_1 - w_1) + \gamma\lambda_2 p_2 q_2' \sin (u_2 - w_2)\} / / (\gamma p_2^{-3} - p_1^{-3}), \tag{33}$$

and one should assume that $i' = 0$, $u'_1 = w' + \tilde{A}_1$, $\tilde{u}_2' = u_2' = w' + \tilde{A}_2$. at $i''_2 \sin(u_2 - \Omega_2) = 0$. Coefficient w' is equal to

$$w' = [B_1 p_1^{-4} (p_1 - p) + B_2 p_2^{-4} (p - \gamma p_2) - B] / (\gamma p_2^{-3} - p_1^{-3}), \tag{34}$$

$$B = \sum_{j=1}^{2} \left[\frac{3}{4} \pi p p_j p_1^{-1} p_2^{-1} \cos (u_j - w_j) - 2 (-1)^j \sin (u_j - w_j)\right] q_j' / p_j^4. \tag{35}$$

One should select from the two derived steady-state solutions ($i' > 0$, $i' < 0$) that to which the smaller value of g corresponds and, accordingly, the local minimum of the characteristic velocity. We find most of these optimal solutions by selecting all the longitudes of the take-off point, determined by formula (31).

Comment 1. Minimum-fuel optimal maneuvers generally occur outside the plane of the initial orbit in the case of noncoplanar circular orbits ($q_1 = q_2 = 0$) in the problem with regard to the time of motion along the orbits, unlike the problem with free time. The same characteristic rate of maneuvering corresponds to the above two types of steady-state solutions ($i' > 0$), $i' < 0$) within accepted accuracy.

Comment 2. All the steady-state solutions of the coplanar problem of a minimum-fuel optimal flight between slightly elliptical boundary orbits of independent interest, with opposite direction of motion and with regard to the time of motion, were found at $i_2 = \pi$ ($\gamma = -1$).

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ON EFFECT OF PARAMETERS OF ACOUSTO-OPTIC DEFLECTOR ON CHARACTERISTICS OF HOLOGRAPHIC MEMORY

907F0352A Frunze IZVESTIYA AKADEMII NAUK KIRGIZSKOY SSR: FIZIKO-TEKHNICHESKYE I MATEMATICHESKIYE NAUKI in Russian No 4, 1989 (manuscript received 7 Apr 88) pp 24-29

[Article by R. T. Seydakhmatova, Institute of Physics, Kirgiz SSR Academy of Sciences]

[Text] It is known that the problem of the handling signal in the main memory (OZU) means selection of one memory element (EP), since the RAM is characterized by sequential sampling of the memory element. Simultaneous sampling of more than one memory element requires several address registers (RA). A model of a holographic main memory (GOZU) is represented in [1] by a graph whose structure corresponds to a tree G, and delivery of an access signal in the model results in activation of only one path of the tree G. If the state of the holographic RAM is understood as the weights that determine the sides of graph G [1], it is obvious that the model encompasses all states of the holographic RAM, inverted in one plane, i.e., a plane and redundant model. This means that the model permits one to select several memory elements in the access cycle; the weights applied to the edges of graph G guarantee sampling of one memory element in the model.

It is necessary to apply the physical interpretation to the weights on the basis of analyzing the operation of the elements of the object beam shaping channel and to the quality of the described and read data.

To do this, let us consider the digital method of design of holograms. It is not redundant, since it formulates the process of writing and sampling of each memory element sequentially. The digital method is described in [2]. Analysis showed that the model of design of the luminous flux passing through the memory element shaping channel does not simulate the operation of the address buses of the memory element, as which such RAM elements as the address register, acoustic frequency shaper and acousto-optic (AO) deflector emerge. The angle of deflection of the light beam is determined by the formula:

$$2\alpha = \frac{\lambda_0}{\lambda_{\text{Л}}}, \qquad (1)$$

where λ_0 is the wavelength of the light beam and $\lambda_{\text{Л}}$ is the acoustic wavelength.

The further influence of $\lambda_{\text{Л}}$ on the parameters of the remaining holographic RAM devices is disregarded.

Let us consider the effect of $\lambda_{\text{Л}}$ and $f_{\text{Л}}$--the frequency of the acoustic wave--on the parameters of the light beam of an intersecting device of the holographic RAM. The memory element is sampled by deflection of the light beam using acoustic waves. The address of the memory element, given in the address register, is converted to an acoustic wave of frequency $f_{\text{Л}}$, which is fed to the input of the cell of the acousto-optic deflector and shapes the combination of diffraction gratings in the cell. The optical beam intersects a deflector, diffracts it in the gratings, and is deflected by an angle determined by frequency $f_{\text{Л}}$. The most effective deflection is obtained when the Bragg condition is fulfilled.

a. Let a beam of light waves, coming from a light source, be described by the formula

$$f_1 = f_0 exp(i(\omega_0 t - K_0 x)). \qquad (2)$$

The acousto-optic deflector deflects it along axis X by angle

$$2\lambda_{\text{Л}} \sin\alpha_x = \lambda_0, \quad \sin\alpha_x = \frac{\lambda_0}{2\lambda_{\text{Л}}}, \quad \alpha_y \cong \frac{\lambda_0}{2\lambda_{\text{Л}}}$$

It can be represented with regard to [3] as:

$$\sin\alpha_m = \sin\alpha + m\,\frac{\lambda_0}{n_0 \lambda_{\text{Л}}}.$$

$$\sin\alpha = \sin\alpha_m - m\,\frac{\lambda_0}{\lambda_{\text{Л}}},$$

where m is the order of diffraction and n_0 is the refractive index of the medium of the acousto-optic deflector, and (2) can then be rewritten as follows:

$$f_{sm}=f_{0m}exp(i(\omega_m t-\kappa_m x)), \quad (3)$$

where $\omega_m=\omega_0+\omega\omega_z$

If we denote $w = w_0{}^x$, we find $\omega_m=\omega_0+\omega_0{}^x\cdot\omega_z$, $\omega_m=\omega_0+\Delta\omega_{0x}$, $\Delta\omega_{0x}=\omega_0{}^x\cdot\omega_z$

The light beam at the output of the cell of the acousto-optic deflector, which is deflected along axis Y, can be represented in the following manner:

$$f_s=f_0exp(i\omega_{0xy}+\kappa_y y), \quad (4)$$

where

$$\omega_{0xy}=\omega_0+(\Delta\omega_x+\Delta\omega_y). \quad (5)$$

It is shown in [3] that waves diffracted on the gratings of the deflector are described at the output by the expression:

$$E_x=\sum E_0(x)exp(i\varphi_x)exp(-i(\omega_m t+\kappa_x x)).$$

If one assumes that the angle of deflection α is proportional to (ω_x + + $\Delta\omega_{0x}$), i.e.,

$$\omega_0+\omega_0{}^x\omega_z\longleftrightarrow\alpha_x-\frac{m}{n_0}\alpha_x,$$

$$\Delta\omega_0=\omega_0{}^x\omega_z.$$

$$\omega_0{}^x\omega_z=-\frac{m}{n_0}\alpha_x,$$

$$\omega_0{}^x=-\frac{m\alpha x}{n_0\omega_z};\quad \alpha_x\longleftrightarrow\omega_0.$$

$$\omega_0{}^x=-\frac{m\omega_0}{n_0\omega_z};\quad \Delta\omega_{0x}=-\frac{m\cdot\omega_0\cdot\omega_z}{n_0\cdot\omega_z}=-\frac{m}{n_0}\omega_0.$$

$$\Delta\omega_{0xy}=\omega_0+(\Delta\omega_{0x}+\Delta\omega_{0y})\longleftrightarrow\alpha_{xy}.$$

Accordingly, (4) permits one to take into account variation of a_x by Δa_x.

b. The value of a_x can also be varied due to variation of the properties of the cell of the acousto-optic deflector as a result of heating.

One of the main parameters of the acousto-optic deflector is $v_Л'$--the propagation velocity of acoustic waves in the deflector. It is known that $v_Л'$ for hard materials is determined by the expression [4]:

$$v'_Л = \sqrt{\frac{E}{\rho} - \left(\frac{4(1+G)\eta}{3}\right)^2 \frac{\pi^2}{\lambda_Л^2}},$$

where η is the viscosity factor, η/η is the kinetic viscosity factor, G is the transverse compression coefficient, and E is Young's modulus.

The value of $v_Л'$ - const for different $f_Л$ is given upon agreement of external and internal conditions. Variation of $v_Л'$ as a result of temperature heating of the medium can be taken into account by ϵ--the loss factor determined by the expression [4]:

$$\eta = \frac{\epsilon \cdot 3}{8\pi f_Л(1+G)}, \quad \epsilon = \frac{\eta 8\pi f_Л(1+G)}{3};$$

then

$$v_Л' = \sqrt{\frac{E}{\rho} - \frac{\epsilon^2}{4\rho^2 f_Л^2 \lambda_Л^2}}$$

is the coefficient of absorption $\alpha = \frac{\pi\epsilon}{\lambda_Л}$, if $\epsilon = 0$, than $v'_Л = \sqrt{\frac{E}{\rho}}$, and if $\epsilon \neq 0$, then

$$(v_Л')^2 = \frac{E}{\rho} - \frac{\epsilon^2}{4\rho^2 v_Л^2}$$

$$\epsilon = 4v_{\text{д}}\sqrt{\rho E - (v_{\text{д}}')^2\rho^2}$$

$$\alpha = \frac{\pi\epsilon}{\lambda_{\text{д}}} = \frac{4\pi v_{\text{д}}\sqrt{\rho E - (v_{\text{д}}')^2\rho^2}}{\lambda_{\text{д}}}, \tag{6}$$

$$\epsilon = \frac{1}{M}, \quad \alpha = \frac{\pi M}{\lambda_{\text{д}}},$$

where M is the Q-factor of the material. The parameters of the materials of the cells of an acousto-optic deflector are given in the table.

Properties of Material To Be Used as Cell of Acousto-Optic Deflector (Crystalline)

(1) Материал	(2) λ, мкм	(3) v м/с	(4) ДБ/см. гц	(5) Мс³/кг	n	ρ²/см³
		(6) Кристаллы				
$LiNbO_3$	0.633	6.57	0,15	7,0	2.200	4.64
$B_{0,76}Nb_2O_6$	0.633	5.50	4	38,6	2.299	5,40
$PbMoO_4$	0.633	3.63	15	36.3	2,262	6,95
TeO_2	0.633	4,20	15	34,5	2,260	6,00

KEY:

1. Material
2. μm
3. m/s
4. DB/cm·Hz
5. Ms³/kg
6. Crystals

Equation (4) can then be represented as follows:

$$f_3 = f_0 exp(-i(\omega_{0xy} + \kappa_{0xy}))exp(-\alpha(x+y)),$$
$$f_3 = f_0 exp(-i(\omega_{0xy} + \kappa_y y))exp(-\alpha(x+y)) \tag{7}$$

c. Light beam (7) intersects the array of a controlled transparency (UT). The light beam diffracted on the controlled transparency can be described as follows [2]:

$$t(x,y) = \sum_{n=1}^{M_I} \sum_{m=1}^{Q_I} g(m,n) circ(2r_1) exp(-i\omega_{0xy} t) exp(-\alpha(x+y))$$

$$r_1 = \frac{1}{d_I} \sqrt{\left(x_I - \left(n - \frac{M_I+1}{2}\right) q_I \right)^2 + \left(y_I - \left(m - \frac{Q_I+1}{2}\right) q_I \right)^2}.$$

where q_I is the distance between the centers bit on the cortrolled transparency, d_I is the diameter of bit, $M_I (Q_I)$ is the number of columns (lines) of the controlled transparency, and g(m, n) = 0V1.

This light beam is confirmed by using direct Fourier transform lenses, while the resulting interference pattern is recorded by the memory (ZS). The intensity distribution in the memory plane is subordinate to the expression [2]:

$$I_H = R^2 \left[1 - \frac{\pi^2}{2} \left(\frac{A d_I^{\ 2}}{2\lambda_{0xy} f_{0xy}} \Lambda_I \left(\pi \sqrt{x_H^2 + y_H^2} \right) \sin(2\pi\mu y_H) \times \right.\right.$$
$$\times \frac{\sin(\pi\beta_I M_I x_H)}{\sin(\pi\beta_I x_H)} \cdot \frac{\sin(\pi\beta_I M_I y_H)}{\sin(\pi\beta_I y_H)}. \qquad (7')$$

In this case, the spatial frequency band in the memory plane is determined by the frequency of the object and reference wave, i.e.,

$$f_d = \frac{3 M_I q_I}{2\lambda_0} + \frac{M_I q_I}{2\lambda_0 f_0} = 4v_{I max}.$$

Then with regard to (5), we find

$$\omega_{0xy} = 2\pi f_{0xy}, f_d = \frac{2 M_I q_I}{\lambda_{0xy} \cdot f_{0xy}} \qquad (8)$$

It was determined in [5] that the power of the luminous flux arriving at the memory for local heating $M_n B_i$ of the film to Curie point T_c, is equal to

$$P = \frac{Q}{\tau_H} = \frac{4\pi K h (T_c - T_a)}{d^2},$$

where T_a is the ambient temperature, h is the thickness of the film, K is the specific thermal conductivity of the film, and d is the period of arrangement of the interference bands; if $h^2 \ll \frac{d^2}{4\pi^2}$, then $P \leftarrow\rightarrow I_0 - I_H$, where I_0 is the light intensity on the surface of the substrate and I_H is the light intensity from the direction of the substrate:

$$I = \frac{4\pi Kh}{d^2}(T_c - T_a)(1 - exp(-\alpha h)),\ I = I_0 \pm \frac{I_\sim}{2}, \tag{9}$$

since $I = I_0 \cdot \cos\,(w_0 t - \kappa x)$. It is known that m--the temperature modulation coefficient of the medium--lies in the range

$$\frac{T_c - T_0}{T_0 - T_a} \leqslant m \leqslant \frac{T_d - T_0}{T_0 - T_a}. \tag{10}$$

One can find from the left half

$$T_c - T_0 = m(T_0 - T_a). \tag{11}$$

where T_0 is the mean value of the temperature in the memory plane. $T_C - T_0$ characterizes the level of excess of T_C over T_0, i.e., it is equal to $I \sim /2 + I_0$, since T_0 is determined by I_0. Accordingly, if d is the period of arrangement of the interference bands in the memory plane, with regard to (8)

$$fd = \frac{1}{d}, \qquad d = \frac{\lambda_{0xy} f_{0xy}}{2M_1 Q_1}. \tag{12}$$

If (12) is substituted into (9), we find

$$I = \frac{2 \cdot 4\pi^2 KhM_1 Q_1}{\lambda_{0xy} f_{0xy}}(T_c - T_a)(1 - exp(-\alpha h)). \tag{13}$$

Let us consider the left side of (10), let us replace the inequality by an equality, and let us take into account the right side in the following manner: the upper value of t in the memory plane is limited by T_d (the breakdown temperature of the medium), and, accordingly, one can change from T_c to T_d, and then

$$T_c - T_a = m(T_0 - T_a) + (T_0 - T_a).$$

Let us substitute the derived expression into (13) and we find:

$$I = \frac{8\pi^2 K h M_1 q_1}{\lambda_{0xy} f_{0xy}} [m(T_0 - T_a) + (T_0 - T_a)](-exp(-\alpha h)) \qquad (14)$$

Accordingly, the distribution of t in the memory plane is dependent on the parameters of the deflector, controlled transparency and the memory. This means that the variation of the memory parameters can be determined by variation of the frequency spectrum.

Based on the foregoing, one can reach the following conclusions. The coefficient of absorption α contained in formula (14) can be expressed by (6), while the angle of deflection α (formula 1) can be expressed by (7).

This means that it has become possible for the first time to determine the effect of the parameters of object beam shaping channel devices on such output parameters of a holographic RAM as P--the power of the light beam or I--the intensity of the light beam. It becomes possible to analyze formula (7′) from the viewpoint of studying the effect of the parameters of a holographic RAM on variation of the values of (I_H).

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